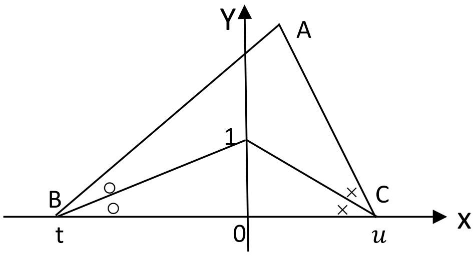
三角形の心　直交座標

右三角形ABCにおいて



点	x 座標	y 座標
頂点 A	$\dfrac{t+u}{tu+1}$	$\dfrac{2tu}{tu+1}$
頂点 B	t	0
頂点 C	u	0
重心 G	$\dfrac{(t+u)(tu+2)}{3(tu+1)}$	$\dfrac{2tu}{3(tu+1)}$
垂心 H	$\dfrac{t+u}{tu+1}$	$\dfrac{-(t-1)(t+1)(u-1)(u+1)}{2(tu+1)}$
外心 O	$\dfrac{t+u}{2}$	$\dfrac{(tu-t+u+1)(tu+t-u+1)}{4(tu+1)}$
内心 I	0	1
傍心 J_A	$t+u$	tu
傍心 J_B	$\dfrac{t(u^2+1)}{tu+1}$	$\dfrac{u(t-u)}{tu+1}$
傍心 J_C	$\dfrac{u(t^2+1)}{tu+1}$	$\dfrac{-t(t-u)}{tu+1}$
円 I とBCの接点	0	0
円 I とABの接点	$\dfrac{2t}{t^2+1}$	$\dfrac{2t^2}{t^2+1}$
円 I とACの接点	$\dfrac{2u}{u^2+1}$	$\dfrac{2u^2}{u^2+1}$
円 J_AとBCの接点	$t+u$	0
円 J_AとABの接点	$\dfrac{t^2(t-u)+t+u}{t^2+1}$	$\dfrac{2tu}{t^2+1}$
円 J_AとACの接点	$\dfrac{-u^2(t-u)+t+u}{u^2+1}$	$\dfrac{2tu}{u^2+1}$
円 J_BとBCの接点	$\dfrac{t(u^2+1)}{tu+1}$	0
円 J_BとABの接点	$\dfrac{t(t^2u^2+t^2+2tu-u^2+1)}{(t^2+1)(tu+1)}$	$\dfrac{2t^2u(t-u)}{(t^2+1)(tu+1)}$
円 J_BとACの接点	$\dfrac{u^3(tu+2)+t}{(u^2+1)(tu+1)}$	$\dfrac{2u(t-u)}{(u^2+1)(tu+1)}$
円 J_CとBCの接点	$\dfrac{u(t^2+1)}{tu+1}$	0
円 J_CとABの接点	$\dfrac{t^3(tu+2)+u}{(t^2+1)(tu+1)}$	$\dfrac{-2t(t-u)}{(t^2+1)(tu+1)}$
円 J_CとACの接点	$\dfrac{u(t^2u^2-t^2+2tu+u^2+1)}{(u^2+1)(tu+1)}$	$\dfrac{-2tu^2(t-u)}{(u^2+1)(tu+1)}$

<table>
<tr><th>点</th><th>x 座標</th><th>y 座標</th></tr>
<tr><td>ド･ロンシャン点
Lo</td><td>$\frac{tu(t+u)}{tu+1}$</td><td>$\frac{t^2u^2-(t-u)^2+1}{tu+1}$</td></tr>
<tr><td>ジェルゴンヌ点
Ge</td><td>$\frac{tu(t+u)}{t^2u^2+t^2-tu+u^2}$</td><td>$\frac{2t^2u^2}{t^2u^2+t^2-tu+u^2}$</td></tr>
<tr><td>ミッテンプンクト
Mi</td><td>$\frac{(t+u)\{tu(t^2u^2+t^2+u^2)+2t^2-3tu+2u^2\}}{2(tu+1)(t^2u^2+t^2-tu+u^2)}$</td><td>$\frac{tu(t-u)^2}{(tu+1)(t^2u^2+t^2-tu+u^2)}$</td></tr>
<tr><td>シュピーカー中心
Sp</td><td>$\frac{(t+u)(tu+2)}{2(tu+1)}$</td><td>$\frac{tu-1}{2(tu+1)}$</td></tr>
<tr><td>ルモアーヌ点
Lu</td><td>$\frac{(t+u)\{t^3u^3+tu(t^2+u^2)+2t^2-3tu+2u^2\}}{2\{(t^2u^2+1)(t^2-tu+u^2)+tu(t^2+u^2)\}}$</td><td>$\frac{tu(t-u)^2(tu+1)}{(t^2u^2+1)(t^2-tu+u^2)+tu(t^2+u^2)}$</td></tr>
<tr><td>フェルマー点 1
F1</td><td colspan="2">$x=\frac{(t+u)\{\sqrt{3}t^3u^3-2t^2u^2(t-u)+\sqrt{3}(t^2+u^2)-10tu(t-u)+\sqrt{3}(2t^2-3tu+2u^2)\}}{2\sqrt{3}\{(t^2-tu+u^2)(t^2u^2+1)-2\sqrt{3}tu(t-u)(tu+1)+tu(t^2+u^2)\}}$
$y=\frac{tu(t-u)(\sqrt{3}t^2-2t-\sqrt{3})(\sqrt{3}u^2+2u-\sqrt{3})}{2\sqrt{3}\{(t^2-tu+u^2)(t^2u^2+1)-2\sqrt{3}tu(t-u)(tu+1)+tu(t^2+u^2)\}}$</td></tr>
<tr><td>フェルマー点 2
F2</td><td colspan="2">$x=\frac{(t+u)\{\sqrt{3}t^3u^3+2t^2u^2(t-u)+\sqrt{3}(t^2+u^2)+10tu(t-u)+\sqrt{3}(2t^2-3tu+2u^2)\}}{2\sqrt{3}\{(t^2-tu+u^2)(t^2u^2+1)+2\sqrt{3}tu(t-u)(tu+1)+tu(t^2+u^2)\}}$
$y=\frac{-tu(t-u)(\sqrt{3}t^2+2t-\sqrt{3})(\sqrt{3}u^2-2u-\sqrt{3})}{2\sqrt{3}\{(t^2-tu+u^2)(t^2u^2+1)+2\sqrt{3}tu(t-u)(tu+1)+tu(t^2+u^2)\}}$</td></tr>
<tr><td>ナポレオン点 1
N1</td><td colspan="2">$x=\frac{(t+u)\{\sqrt{3}t^3u^3-6t^2u^2(t-u)+\sqrt{3}(t^2+u^2)-14tu(t-u)+\sqrt{3}(2t^2-3tu+2u^2)\}}{2\{\sqrt{3}(t^2-tu+u^2)(t^2u^2+1)-10tu(t-u)(tu+1)+\sqrt{3}tu(t^2+u^2)\}}$
$y=\frac{tu(t-u)(t^2-2\sqrt{3}t-1)(u^2+2\sqrt{3}u-1)}{2\{\sqrt{3}(t^2-tu+u^2)(t^2u^2+1)-10tu(t-u)(tu+1)+\sqrt{3}tu(t^2+u^2)\}}$</td></tr>
<tr><td>ナポレオン点 2
N2</td><td colspan="2">$x=\frac{(t+u)\{\sqrt{3}t^3u^3+6t^2u^2(t-u)+\sqrt{3}(t^2+u^2)+14tu(t-u)+\sqrt{3}(2t^2-3tu+2u^2)\}}{2\{\sqrt{3}(t^2-tu+u^2)(t^2u^2+1)+10tu(t-u)(tu+1)+\sqrt{3}tu(t^2+u^2)\}}$
$y=\frac{-tu(t-u)(t^2+2\sqrt{3}t-1)(u^2-2\sqrt{3}u-1)}{2\sqrt{3}\{\sqrt{3}(t^2-tu+u^2)(t^2u^2+1)+10tu(t-u)(tu+1)+\sqrt{3}tu(t^2+u^2)\}}$</td></tr>
<tr><td>ナーゲル点 Na</td><td>$\frac{(t+u)(tu+2)}{tu+1}$</td><td>$\frac{-2}{tu+1}$</td></tr>
<tr><td>ベバン点 Be</td><td>$t+u$</td><td>$\frac{t^2u^2-(t-u)^2-1}{2(tu+1)}$</td></tr>
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2024.03　松戸市　広川